Evaluate the following integrals:

We will freely use the following results:

which are proved in the documents “ln(sinx) and related integrals” and “frac{lnx}{x^2+1} and related integrals” respectively.

*Remark: Originally I had set out to solve , but through manipulations, substitutions, and integration by parts, I stumbled into the other integrals, mainly , and . However since these integrals are pleasing in their own right, they will be solved here as well.*

Before embarking on the solution, we will explain why differentiation under the integral sign cannot be used to solve even though defining functions (noting or (noting ) seems natural. Focusing on , differentiating under the integral eventually leads to and the term (which appears because is a bound of integration) has no good closed-form antiderivative. Even substituting prior to differentiating under the integral will not work as will still end up being a bound of integration. For the same reason, will not work either. Differentiating under the integral would have worked if the bounds of integration were and .

Although was rather inaccessible, I was eventually able to solve . We have:

But by substituting we see that , so

However we also have

Solving simultaneously with eventually yields

From here, the other integrals can be solved. Using integration by parts,

Returning to and substituting ,

Lastly, to solve ,